At the time of issue this report has not been published. Any author desiring to quote this paper should refer to it as "A paper of the Meteorological Research Committee (London).
M.R.P. No.965 by Mr. D.H. Johnson.
A copy is available in the Library of the Meteorological Office".

M.R.P.965
S.C.II/207
31st March, 1956

AIR MINISTRY

METEOROLOGICAL RESEARCH COMMITTEE

Objective Analysis

by

D.H. Johnson (Meteorological Office)

1. Introduction

Techniques of numerical forecasting which are being developed in several countries, require the specification of the initial state in terms of the heights of isobaric surfaces and the thicknesses of isobaric layers at a network of points. For experimental numerical forecasts, produced in the Forecast Research Division (1), contour charts were constructed by the thickness technique and values of the contour height were read off from the charts, at the grid points, by interpolating between the isopleths. The time taken for the whole process, including plotting of observations, drawing of charts, interpolation and checking is to be measured in hours and exceeds the time taken by the faster electronic computers to produce a 24-hour forecast. If the initial processing of the data could be performed objectively, by numerical methods adapted for use with electronic machines, an appreciable economy in time might result.

Some elementary experiments in objective analysis are described in this paper. The computations were carried out on the Ferranti Mk. I machine at Manchester University by F.H. Bushby and V.M. Huckle, who also devised the programmes.

2. The problem

The Central Forecasting Office is concerned with a region, over different sectors of which the observations vary greatly in density, frequency, accuracy and homogeneity. A practical scheme of analysis requires a degree of flexibility sufficient to deal with these variations. The system of graphical analysis at present used in routine work enables the following factors to be considered in the construction of a contour chart for a given level:

(i) observations of height and wind for the synoptic hour to which the analysis relates - allowance is made for random errors, systematic errors affecting whole networks and mistakes due to coding, transmission, etc.;

(ii) continuity in time from previous analyses;

(iii) continuity in the vertical through the thickness technique, which not only maintains consistency between levels but also allows the upper air analysis to benefit from the more precise definition of the surface pattern.

In parts of the European network, for example over the British Isles, the observations are so close together that no attention need be paid to items (ii) and (iii) but elsewhere, particularly over the Atlantic, the latter factors are very important. It seems inevitable that a successful scheme of objective analysis must also take into account continuity in time and

/consistency

consistency in the vertical. As a first step, however, attention has been confined to the basic problem of interpolating within a two-dimensional network of observations, without regard to conditions at other levels or at other times.

3. Method of approach

One method of interpolating for the height of a contour surface at a given grid point is to represent the contour height over some area surrounding the grid point mathematically as a function of position, the function being chosen so as to give the appropriate values at the observing stations. By applying the geostrophic relation the conventional analyst is able to take the observed winds into account in spacing and aligning the isopleths of contour height, and it seems reasonable to use winds as well as contour heights to determine the required function.

An attempt was made to represent the contour height, H, as a quadratic function of suitable co-ordinates on the earth's surface. The co-ordinates chosen were those used for the numerical forecasts. They corresponded to rectangular Cartesian co-ordinates on a stereographic orthomorphic projection of the earth's surface from the south pole onto the tangent plane through the north pole. H was taken to be expressible in the form, $H = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ where a, h, b, g, f, c were constants to be determined from the observations. Because the observations contained errors, it was advisable not to fit them exactly but to introduce a degree of smoothing using the principle of least squares. A least-squares solution was obtained by choosing a, h, etc., so as to minimise the expression,

$$\sum_i (H - H_i)^2 + T^2 \sum_i (W - W_i)^2 \qquad \text{(1)}$$

where the H_i were the observed contour heights, the W_i were observed winds, W was the geostrophic wind, T was a weighting factor which decided the relative weight to be given to contour height and wind observations, and the summation was taken over i observing stations. Components of W, V_x and V_y, were given by $-\frac{\beta g}{l}\frac{\partial H}{\partial y}$ and $\frac{\beta g}{l}\frac{\partial H}{\partial x}$ respectively, where g, l have their usual significance and β is a magnification factor arising from the choice of co-ordinates. The second term of (1) could be expanded as $T^2\left\{\sum_i (V-V_i)_x^2 + \sum_i (V-V_i)_y^2\right\}$, the subscripts denoting x and y components. Evidently, each wind observation gives two independent pieces of information regarding the contour gradient. Equation (1) was minimised when a, h, etc. satisfied the six simultaneous linear equations obtained by partial differentiation of expression (1) by a, h, b, g, f, and c, in turn.

4. Choice of weight

The weighting factor, T, which occurs in expression (1), holds a balance between contour heights and winds and depends, amongst other things, upon the units of H and W. It allows the terms $\sum_i (H-H_i)^2$ and $T^2 \sum_i (W-W_i)^2$ to attain the same order of magnitude: if these terms differed greatly, the least squares process would ignore the smaller of the two. In the computations described below it was convenient to work with W in ft./sec. and H in ft. For the two terms of expression (1) to have equivalent dimensions, the units of T have been taken as seconds. The value of $\sum_i (H-H_i)^2$ depends on the random errors of observation of contour height and also upon a kind of topographical error which arises because quadratic surfaces comprise a restricted class of shapes: the true contour surface will not in general coincide with one of these shapes, although there may be a

/sufficient

sufficient resemblance over a limited area. $\sum_i (\mathbb{V} - \mathbb{V}_i)^2$ contains ageostrophic effects as well as errors of observation and errors of a topographical character. The "topographical" errors will vary according to the function used to represent the contour surface, so T itself may require adjustment according to the function chosen.

Computation No. 1

To determine an optimum weighting factor, a quadratic surface was fitted, by the method outlined above, to 500 mb. heights and winds from six British stations, Aldergrove, Hemsby, Crawley, Leuchars, Camborne and Valentia, for 100 occasions selected from the first nine months of 1954. Computations were made in each case for the ten integral values of T from 0 to 9. Occasions were chosen when there were reports of contour height and wind from all six stations, but otherwise the cases were picked at random. To test the value of the quadratic surfaces for interpolation, the heights and geostrophic winds given by the surfaces for Liverpool were compared with those observed by the Liverpool upper air station. Figure 1 shows the disposition of the stations used for these and subsequent calculations. The root-mean-square differences between observed and computed contour heights and winds at Liverpool for the 100 cases, are shown graphically in Figure 2. Values for T = 0 were special since, if the winds are ignored, the six contour heights can be fitted exactly by a quadratic and there is no smoothing. The lowest interpolation errors apparently occurred in the range from T = 1 to T = 2, where the r.m.s. wind difference was 11.4 kt. and the height difference was 52 ft.

Figure 3 shows the variation with T of the r.m.s. height and wind differences when summed over the six stations to which surfaces were fitted and over the 100 cases. In the range T = 1 to T = 2 the r.m.s. height difference rose from 21 to 30 ft. whilst the r.m.s. wind difference fell from 15 to 12 kt. (25 to 20 ft./sec.). So the two terms of expression (1) did in practice attain the same order of magnitude:- at $T = 1$, $\sum (H - H_i)^2 = 400$ ft. while $T^2 \sum (\mathbb{V} - \mathbb{V}_i)^2 = 625$ ft.^{2}, T^2 having units of sec.^{2}; at $T = 2$, $\sum (H - H_i)^2 = 900$ ft.^{2} while $T^2 \sum (\mathbb{V} - \mathbb{V}_i)^2 = 1600$ ft^2.

Computation No. 2

It was remarked above that the results obtained for T = 0 in Computation No. 1 were special since the contour heights at the six stations were exactly fitted. The computations were repeated for 40 of the 100 cases using observations from Stornoway and Lerwick in addition to the six previously used. In this case, even at T = 0, there were more pieces of information than were required to determine the co-efficients of the quadratic uniquely.

In Figure 4 the r.m.s. height and wind differences at Liverpool for the eight station computations are compared with results for the same 40 cases from Computation No. 1. Except for the heights at T = 0, the Computation No. 2 differences exceed those of Computation No. 1. The inclusion of the two extra heights reduced the r.m.s. Liverpool height difference considerably. The improvement here was due entirely to the beneficial effect of smoothing out errors of observation, etc., since results for T = 1 to T = 9 suggest that it was generally undesirable to extend the area over which the quadratic was fitted beyond that covered in Computation No. 1. As in Computation No. 1, values of T in the range from T = 1 to T = 2 appeared to be satisfactory from the point of view of minimising the errors of representation. Neither the wind nor the height errors were very sensitive to variations in T.

Figure 5 gives the standard errors of fitting at the eight stations. These may be compared with the corresponding values for Computation No. 1 in Figure 3. Computation No. 1 values were smaller except for the winds at T = 0.

/Computation

Computation No. 3

To examine whether a surface of greater complexity than the quadratic was of more value for interpolation, a cubic surface was fitted to the six stations used for Computation No. 1, in 38 cases. The method of obtaining the best-fitting surface was similar in principle to that outlined in Section 2 for the quadratic. Computations were made for six values of T: T = 1, $\sqrt{2}$, 2, $2\sqrt{2}$, 4 and 6.

The r.m.s. differences between observed and interpolated heights and winds at Liverpool are given in Figure 6. Neither the height nor the wind differences showed much variation over the range of T explored, but such variations as there were followed a different pattern from the quadratic results of Computation No. 1. The lowest Liverpool wind difference occurred at about T = 4 whereas the height errors increased with T throughout the range examined. The cubic results are best compared with the quadratic six-station values given in Figure 4 which, with two exceptions, refer to the same occasions. The two exceptions do not materially affect the comparison. The cubic surfaces gave a slightly better fit than the quadratic at Liverpool.

Figure 7 gives the r.m.s. errors of representation at the six stations whose observations were used to obtain the cubics. The values are naturally lower than those obtained for the quadratic (Figure 3) since the cubics have a greater degree of freedom.

The cubic results show how the choice of weighting factor T may depend upon the purpose for which the interpolation is required. A preliminary to the preparation of a numerical forecast is the evaluation of the Laplacian derivative of the initial contour field. Normally, the Laplacian is derived from the set of interpolated contour heights by means of the finite difference approximation. If the contour field is represented by some mathematical function, it is possible to obtain the Laplacian in a second way, namely by direct differentiation of the function itself. For this latter purpose it is desirable that the function should not just give an accurate interpolation for height at the grid point, but should also approximate closely to the true contour surface over a certain area surrounding the grid point. In Computation No. 3 the contour height interpolation error was lowest at T = 1 (Figure 6), but Figure 7 shows that at T = 1 the heights at the six stations were fitted with an error of only 10 ft. This closeness of fit was unrealistic since the standard error of measurement of 500 mb. contour height at British stations is 30 ft. Furthermore the winds at the six stations were fitted at T = 1 with an error of about 10 kt., which is larger than the standard error, 3 kt., of British 500 mb. wind observations. At T = 4, on the other hand, the r.m.s. height error at the six stations was 30 ft. and the r.m.s. wind error was 5 kt. Thus, if a realistic fit is required over an area of 300 mi. or so centred upon Liverpool, a weighting factor T = 4 is to be preferred to T = 1.

According to Murray (2) the r.m.s. difference between observed winds and geostrophic winds measured from the C.F.O. working charts is 14 kt. at 500 mb. over the British Isles. It is interesting to note that the wind values interpolated from the cubic surfaces when compared with independent observed winds gave a r.m.s. difference of less than 10 kt. Murray shows that the 14 kt. can be broken down into the following components: 9 kt. due to the ageostrophic wind; 4 kt. due to turbulent wind fluctuations; 3 kt. due to errors in the observed wind; 9 kt. due to errors inherent in the graphical technique. The error of representation of the wind field given by the true contour pattern will contain the first three of these four components and should therefore have a root-mean-square value of about 10 kt. On this basis it appears that the 5 kt. error of representation at the six stations, given by the cubics for a weighting factor T = 4, is lower than is justified, but there is an advantage to be gained in fitting the winds more closely than seems proper on purely geostrophic grounds. Numerical forecasting requires the specification of the initial field of vorticity. The geostrophic

vorticity, deduced from the contour pattern, is used as an approximation to the vorticity of the actual wind field. This approximation is believed to be responsible on occasions for sizeable errors in the forecasts. It may therefore be advantageous if the "geostrophic" wind field deduced from the objective analysis actually incorporates certain ageostrophic effects, and thus gives a closer approximation to the true wind field than the geostrophic wind field implied by the true contour pattern. Such a shrewd objective analysis is possible because the ageostrophic effects (apart from turbulence) are not distributed randomly, but form a sensible pattern of variation on the synoptic scale.

5. Various networks - Computation No. 4

The six-station network, centred upon Liverpool, was chosen somewhat arbitrarily as suitable for use in the above interpolation experiments. To examine the effect on the errors of representation of varying the distribution and quantity of data, quadratic surfaces were fitted to sixteen selections of data from the eight stations used in Computation No. 2. The computations were made with a weighting factor $T = \sqrt{2}$ for each of 40 occasions. The r.m.s. errors of representation at Liverpool and at stations used in the computations are given for each selection in Table 1, which also contains, for comparison, values previously obtained for corresponding occasions in the six-station and eight-station calculations (Computations Nos. 1 and 2).

The first six rows of Table 1 (series B, C, D, E, F and six-station) give information regarding the relative value of contour heights and winds. Comparing the Liverpool interpolation errors, series D, E and F, which contained more heights than winds, improved markedly upon series B and C in which winds preponderated.

Series M, K and I contained wind and contour height observations from five, four and three stations respectively, a compromise being made in the choice of stations between proximity to Liverpool and regularity of distribution. Each of these arrangements gave errors of interpolation similar to those of the six-station data, except that the three-station arrangement (series I) gave a slightly better fit to the Liverpool wind: Liverpool is not far from the centroid of the triangle formed by the three stations, Aldergrove, Hemsby and Camborne. When the size of the triangle was increased (series J and H), the interpolation errors were also increased.

Most of the above selections were chosen so as to have a reasonably symmetric distribution with respect to Liverpool, although comparison between series D and E shows how the disposition of the two winds materially affected the contour height errors. In series N, O and P the data were not symmetrically arranged about Liverpool, in fact for series O and P the problem was one of extrapolation rather than interpolation. In these last three cases the errors of representation were comparatively large.

6. Distance weighting - Computation No. 5

Since the correlation between the winds or contour heights at two places varies inversely with their distance apart, it may be possible to reduce the interpolation errors by weighting the observations according to their distance from the grid point. To examine this possibility a quadratic surface was fitted to the six stations of Computation No. 1 for 100 cases, each observation being weighted by a factor, p, which was a function of distance from Liverpool. The expression to be minimised, in place of expression (1) was,

$$\sum_i p_i (H - H_i)^2 + T^2 \sum_i p_i (W - W_i)^2$$

/Two

Two sets of distance weighting factors were tried. The first weight, p_α, was expressible as a function of distance, r, from Liverpool in the form $1/1+r^3$ and the second weight, p_β, as $1/1+r$. The unit of r was chosen so that for p_α, Valentia observations received about one half of the weight given to Aldergrove observations, and for p_β, Valentia observations received about three-quarters of the weight given to observations from Aldergrove. The weighting factors are shown graphically in Figure 8.

The interpolation errors are given in Table 2. The results were disappointing. Not only were the average errors practically unchanged by distance weighting, but the individual surfaces for the same occasions were nearly identical. Possibly the distance weights were not sufficiently discriminating, but it is more likely that the network of stations was unsuitable for distance weighting experiments, since four of the stations were bunched together on the distance scale (see Figure 8).

7. Oceanic networks

The foregoing experiments gave promising results, but the British network of upper air observations is probably the best in existence for accuracy and density of observations. The worst networks are found over the oceans. A difficulty which arises in experimenting with a weather ship network is that the reservation of one observation for verification purposes alone seriously depletes the network. The alternative to checking the objective interpolations against observations is to compare them with values read from a conventionally analysed chart. This latter practice may be unsound however, particularly for experiments in which "T" is varied, since the conventional analysis is subjective and itself depends upon the analyst's opinion regarding the relative weights to be given to wind and height observations. To avoid these difficulties the five stations, weather ships "I", "J" and "K", Crawley and Stornoway were chosen as being equivalent to a mid-ocean network. Valentia observations were used to test the accuracy of the interpolated values.

Computation No. 6

Quadratic surfaces were fitted to observations from the five stations for 100 occasions chosen from the year 1954. Computations were made for T-values of 1, 2 and 4. The errors of representation appear in Table 3. The interpolation errors considerably exceeded those obtained in previous computations.

Computation No. 7

Computation No. 6 was repeated, replacing the quadratic surfaces by cubics. Table 3 contains the results. The Valentia errors are slightly smaller for the cubics than for the quadratics.

8. Individual cases

On a few occasions the height contours given by the objective analyses have been drawn for visual comparison with the contours produced by analysts at the Central Forecasting Office for routine use. Figures 9 to 15 show the individual analyses. In scrutinising the objective analyses it should be borne in mind that the only stations taken into account in their computation were, Aldergrove, Hemsby, Leuchars, Crawley, Camborne and Valentia (not Liverpool). The objective analyses were not designed to give any sort of a fit outside of the area bounded by these stations.

Figures 9 to 13 contain quadratic analyses for three values of T. It is obvious that as T increased, the fit to the winds at the six stations was improved at the expense of the contour heights. Analyses for T = 0 take no

/account

account of the winds. In Figures 9(a), 12(a) and 13(a) the analyses for T = 0 are quite unrealistic and give high errors of interpolation at Liverpool. The quadratic contours are recognisable as families of conic sections: there is a slight distortion because the projection of the map on which they are drawn is conical, not stereographic.

In Figures 9, 14 and 15 quadratic analyses may be compared with cubic analyses for the same occasions.

9. Comparison with other work

Attempts to represent wind and contour fields by cubic polynomials have been made by Panofsky (3). Cubics were fitted over an area of the order of 10^6 square miles, which is about four times greater than the area considered for Computation No. 1. The intention was ultimately to construct an objective analysis by piecing together several such cubic surfaces. Calculations were not made electronically and few cubics were computed. Winds were weighted by a factor corresponding to T = 2. Visual comparison of the objective analyses with the corresponding subjective analyses showed that the two methods gave similar contour patterns, but the objective analyses were somewhat smoother. The Laplacian derivatives of the contour fields were also computed: here the agreement was "not good".

The last result might have been anticipated since the Laplacian of a cubic is a linear function and experience of vorticity fields computed from subjective analyses suggests that the true vorticity fields are very far from linear; in fact they are usually more complex than the contour patterns themselves. The implication of comparative simplicity of the vorticity field may be a considerable objection to the use of polynomials in representing contour patterns over large areas.

Gilchrist and Cressman (4) electronically computed objective analyses for a grid of 19 x 19 points covering most of North America and a part of the Atlantic. The method was very similar to that used for the computations described in Section 4: a quadratic surface was fitted to observations within a limited distance of a grid point to obtain an interpolated contour height. The process was repeated for each grid point. The computations were made for a single occasion and for a series of values of T. The interpolated heights were compared with the mean of three subjective analyses. The minimum height errors occurred with a wind weighting factor corresponding to T = 2. The finite difference Laplacians were computed for both objective and subjective analyses, the closest agreement being attained at T = 3. Numerical forecasts were made on the basis of both analyses. It is stated that neither forecast was observably superior to the other. The 24-hr. changes in 1000 mb. height, forecast by the two methods, were correlated and gave a correlation co-efficient of ·96. The authors conclude that where a good data coverage is found, e.g., over North America, objective analyses are satisfactory for numerical prediction.

10. Future investigations

The experiments described in Sections 4 to 6 have shown that in a close network of observations, a reasonably accurate objective interpolation for contour height and wind can be made. Comparisons with the errors of representation of the conventional contour analysis, made in Section 4, suggest, as did the work of Gilchrist and Cressman, that in regions of good data coverage, objective analyses will probably be at least as satisfactory as conventional analyses for numerical prediction. However, it was pointed out in Section 2 that only in limited regions would an analysis, which took no account of continuity in time and in the vertical, give good results. Computations described in Section 7 for the ocean network confirmed this expectation.

/There

There is no difficulty in principle in introducing an element of continuity in time into the analysis, for forecast values of height and wind can be taken into account in the least squares process on the same basis as observations. The weight given to a forecast value would naturally be much less than the weight given to an observation.

The object of the present series of computations was to derive interpolated heights and winds for a single grid point. In extending the technique to get interpolations over a network of grid points one is led to consider whether it is better to represent the contour field over the whole network by a single function or whether a fresh function should be derived for each grid point. Gilchrist and Cressman (4) point out that the latter alternative agrees with synoptic practice in considering data only from the immediate vicinity of the grid point in question. For the former alternative, difficulties may arise due to the dependence of the expression for the contour height at a given grid point on data from distant regions. There is also the objection to the use of a single function for representation over a wide area mentioned in Section 9, namely that the degree of complexity of the implied geostrophic vorticity field may be over-restricted, although this objection would be less valid for harmonic functions, for example, than for polynomials. On balance, it seems that the method which treats each grid point independently is likely to prove the more flexible approach and at the present stage there seems no reason for adopting an alternative technique, although other approaches will require investigation in due course.

11. Conclusions

(1) In a close network of reliable observations, objective analyses are capable of representing the 500 mb. winds with a standard error of about 10 kt. This figure compares favourably with the errors of representation, about 14 kt., of the Central Forecasting Office routine analyses for the same area.

(2) In more open networks, such as are found over the Atlantic Ocean, providing that both winds and contour heights are available, winds can be interpolated with a standard error of about 16 or 17 kt. Errors of the conventional analysis are not known for such a region, but they are unlikely to be less than 16 or 17 kt., judging from the data available for the land area.

(3) The optimum weight to be given to wind observations as compared with contour heights varies according to the nature of the interpolation. When interpolating for wind or contour height using quadratic surfaces the weighting factor is best chosen so that a discrepancy of 30 ft. in contour height receives the same weight as a discrepancy of 10 - 15 kt. in wind. Using cubic surfaces, a similar weight gives good results in interpolation for contour height, but the optimum weight for wind interpolation equates a discrepancy of 30 ft. in contour height to a discrepancy of 5 kt. in wind.

(4) The errors of interpolation are not very sensitive either to small variations in the relative weights given to contour heights and winds or to small variations in the number and disposition of the observations.

(5) Tests to determine the effect of weighting observations according to their distance from the grid-point at which an interpolation is required are considered inconclusive and it is recommended that a further test be made with a less regular network of observations.

/(6)

(6) There is ample justification in results so far obtained for further work designed to produce interpolated contour heights at a network of points over the whole area from which the initial data must be taken for numerical forecasting. These experiments should include methods of maintaining continuity in time and consistency between levels.

12. References

(1)	Bushby, F.H. and Hinds, M.K.	1954	Quart. J. R. Met. Soc., Vol. 80, p. 165.
(2)	Murray, R.	1954	Met. Office, London, Professional Notes, Vol. 7, No. 110.
(3)	Panofsky, H.A.	1949	J. Met., Vol. 6, p. 386.
(4)	Gilchrist, B. and Cressman, G.P.	1954	Tellus, Vol. 6, p. 309.

Table 1. Computation No. 4. Errors of representation for eighteen selections of data

Series	Selection																R.m.s. errors of representation			
	Heights								Winds								At Liverpool		At selected stations	
	Ald.	Hem.	Leu.	Cra.	Cam.	Val.	Sto.	Ler.	Ald.	Hem.	Leu.	Cra.	Cam.	Val.	Sto.	Ler.	Wind (kt.)	Height (ft.)	Wind (kt.)	Height (ft.)
B	1								1	1	1						14·0	74	4	0
C	1								1	1	1	1	1	1			14·5	76	9	0
D	1	1	1	1	1	1			1					1			11·2	64	12	20
E	1	1	1	1	1	1			1	1							11·5	53	13	19
F	1	1	1	1	1	1			1	1	1	1					11·5	53	12	21
6-station	1	1	1	1	1	1			1	1	1	1	1	1			11·5	50	14	25
8-station	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	12·8	64	14	30
M	1	1	1	1	1				1	1	1	1	1				11·4	54	12	18
L		1	1		1	1				1	1		1	1			11·9	58	14	14
K	1	1	1		1				1	1	1		1				11·4	55	12	13
J			1	1		1					1	1		1			12·8	69	11	9
I	1	1			1				1	1			1				10·1	52	9	10
A	1	1			1				1	1							10·4	51	8	6
H				1		1		1				1		1		1	14·4	81	14	8
N	1	1	1			1	1		1	1	1			1	1		12·3	62	14	18
O	1		1		1	1	1		1		1		1	1	1		13·6	75	13	19
P	1		1				1	1	1		1				1	1	20·6	67	10	17

Table 2. Computation No. 5

Errors of representation for a quadratic surface using distance weighting factors

Distance weighting factors	p_α			p_β			None		
Value of T sec.	1	$\sqrt{2}$	$2\sqrt{2}$	1	$\sqrt{2}$	$2\sqrt{2}$	1	$\sqrt{2}$ *	$2\sqrt{2}$ *
R.m.s. wind error at Liverpool (kt.)	11·3	11·3	11·8	11·3	11·3	11·8	11·3	?	11.8
R.m.s. height error at Liverpool (ft.)	51	51	51	52	51	52	52	52	53
R.m.s. wind error at the 6 stations (kt.)	15	14	11	15	14	11	15	14	11
R.m.s. height error at the 6 stations (ft.)	22	26	38	22	25	37	22	25	36

* Values interpolated from Figures 2 and 3. Interpolation doubtful for r.m.s. wind error at Liverpool at T = $\sqrt{2}$.

Table 3. Computations Nos. 6 and 7

Errors of representation for quadratic and cubic interpolation using an ocean-type network

Nature of surface	Quadratic (Comp. 6)			Cubic (Comp. 7)		
Values of T sec.	1	2	4	1	2	4
R.m.s. wind error at Valentia (kt.)	18	18	19	17	17	16
R.m.s. height error at Valentia (ft.)	96	94	94	92	92	92
R.m.s. wind error at the 5 stations (kt.)	25	23	21	16	14	11
R.m.s. height error at the 5 stations (ft.)	11	30	62	6	19	46

Fig. 1
Key to stations used in the computations

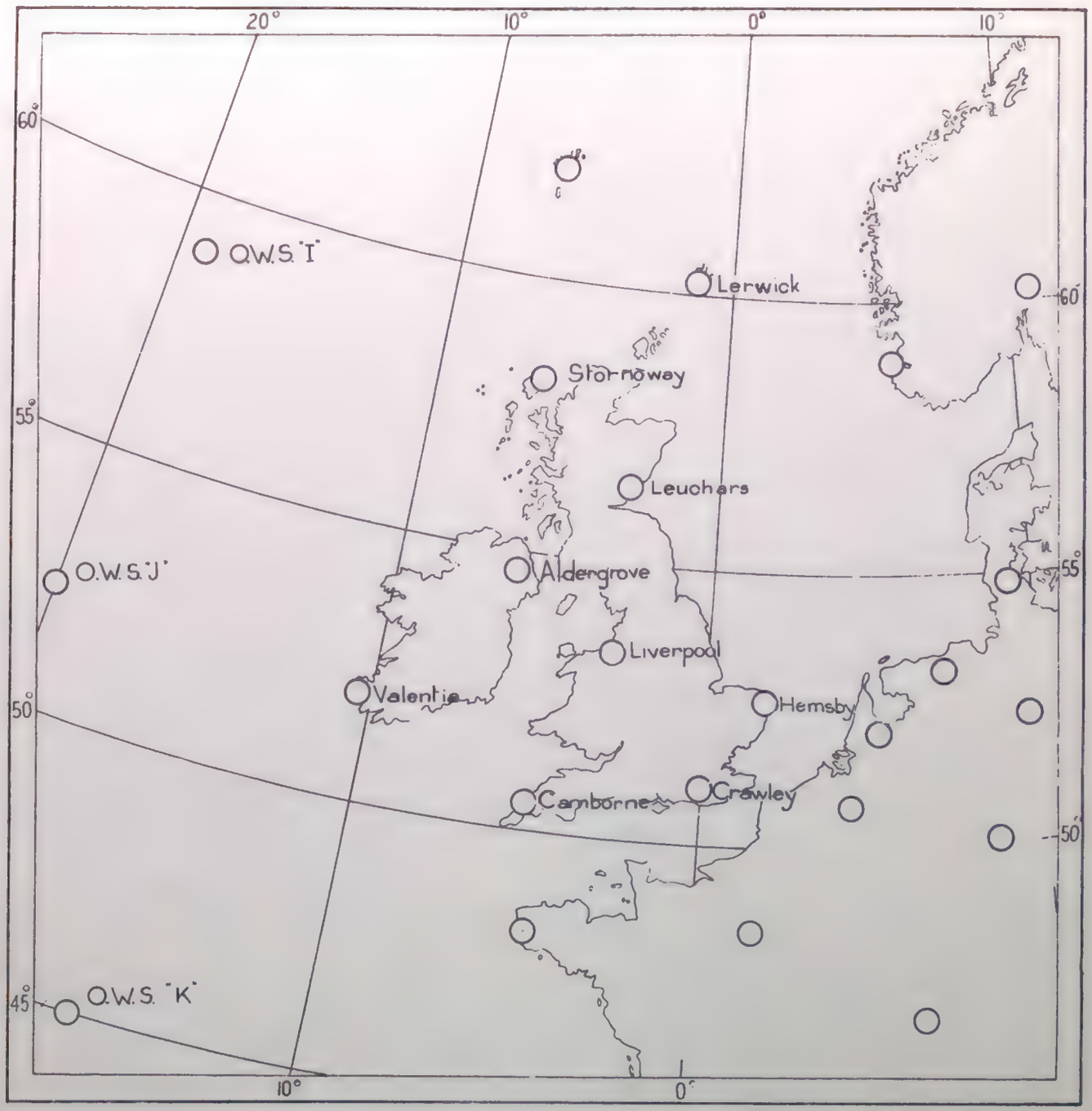


Fig.2

COMPUTATION No.1

Errors of representation at Liverpool

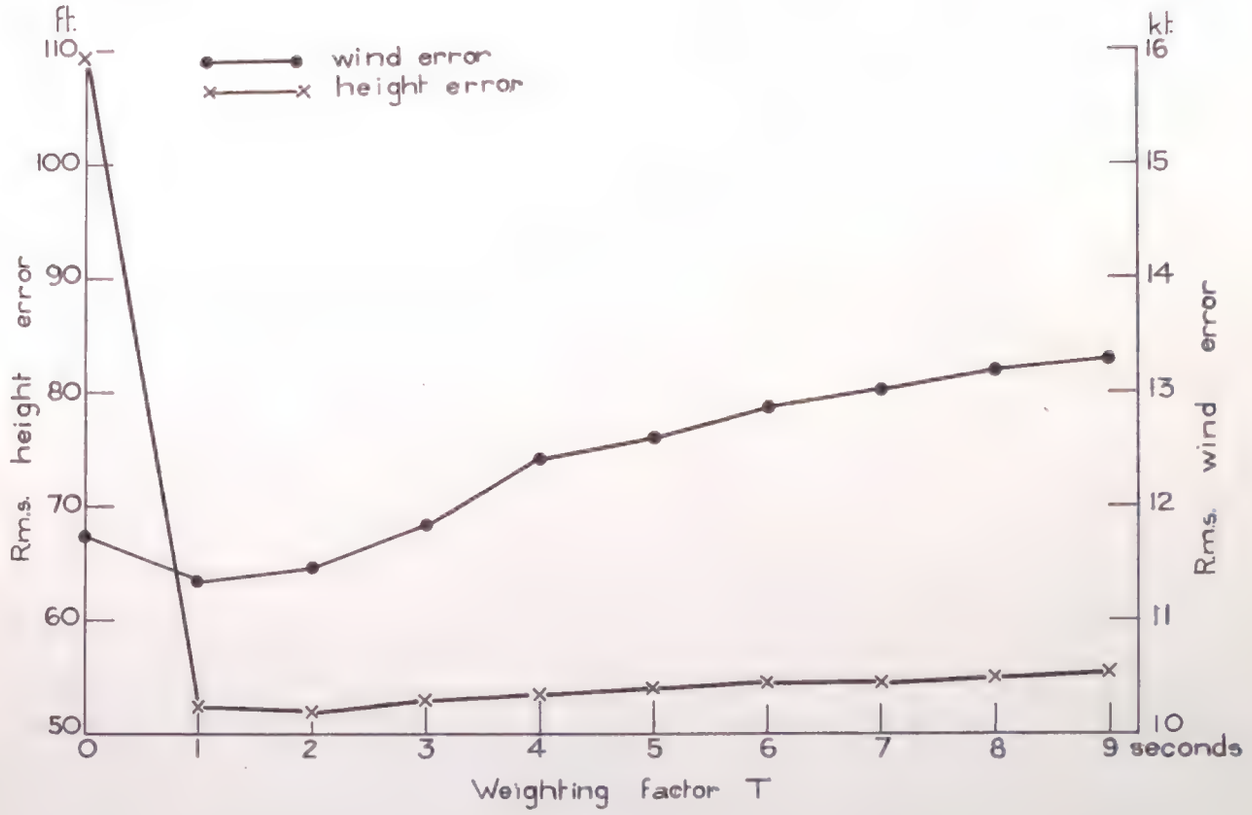


Fig. 3

COMPUTATION No. 1

Errors of representation at the six stations

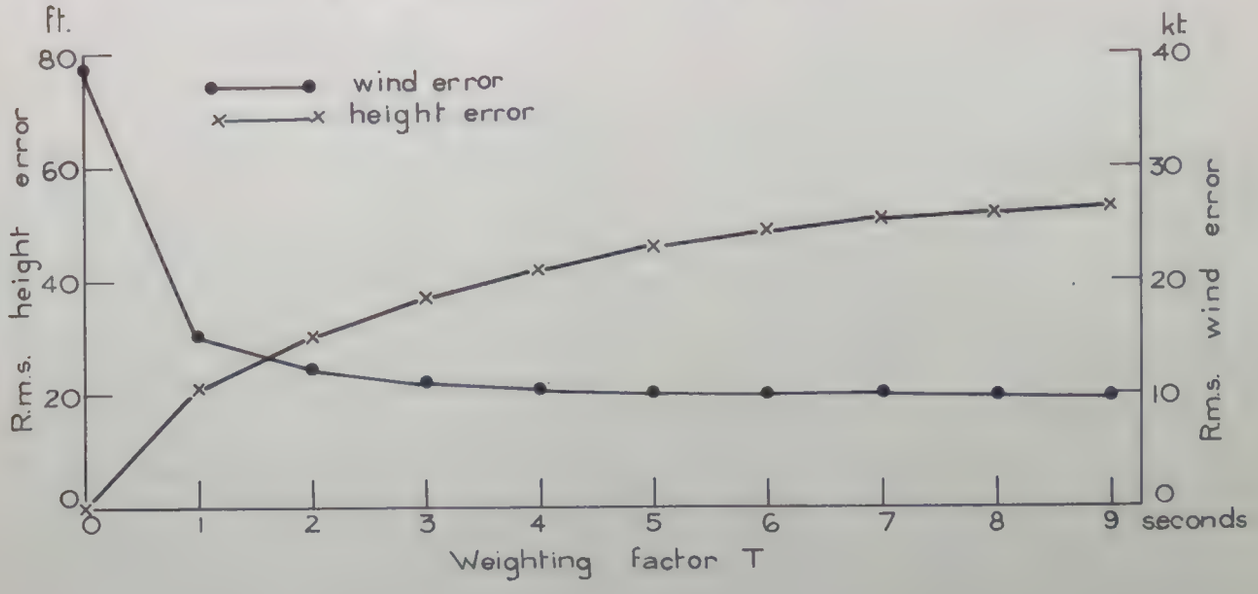


Fig. 4
COMPUTATION No. 2
Errors of representation at Liverpool

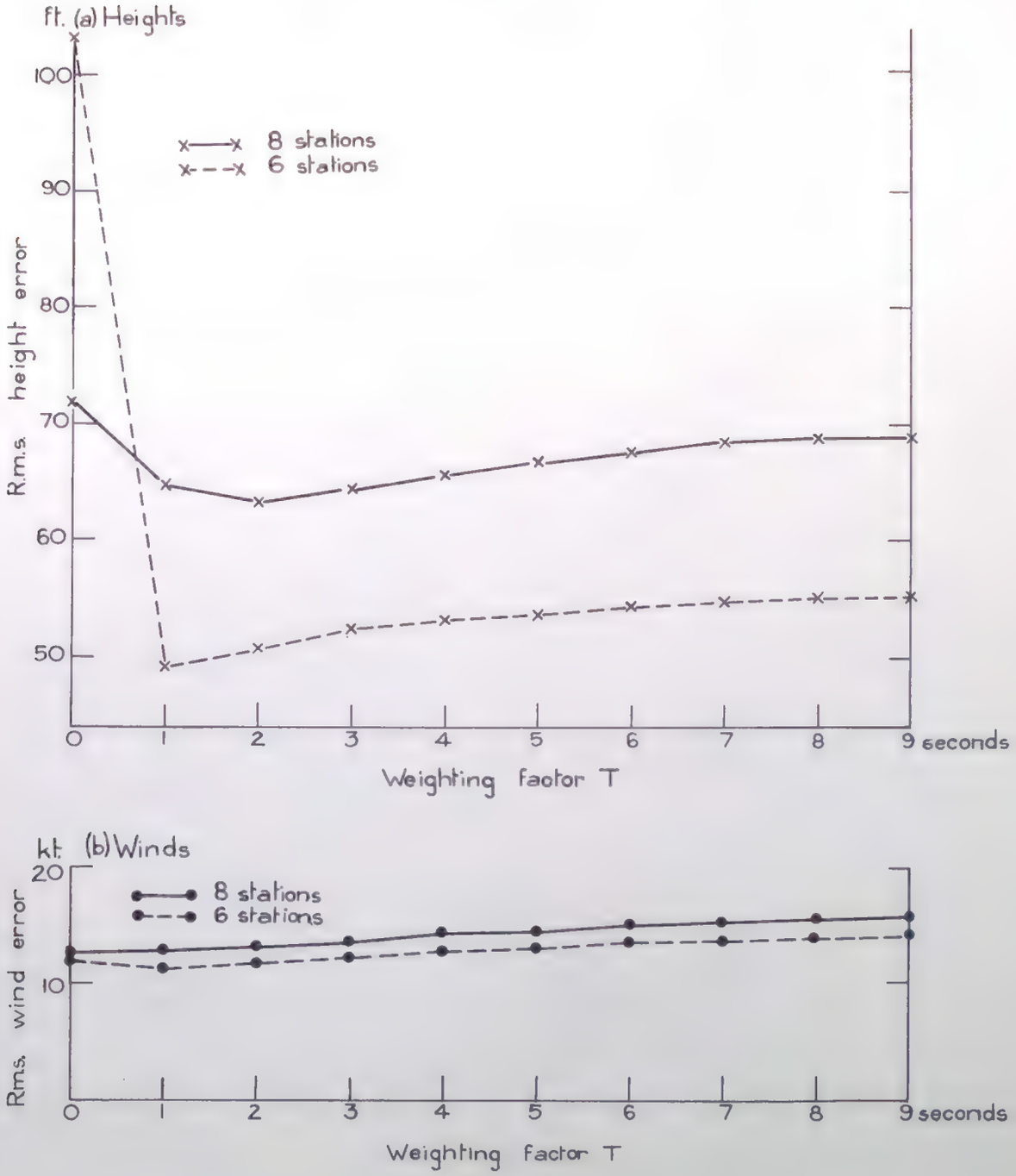


Fig. 5
COMPUTATION No. 2
Errors of representation at the eight stations

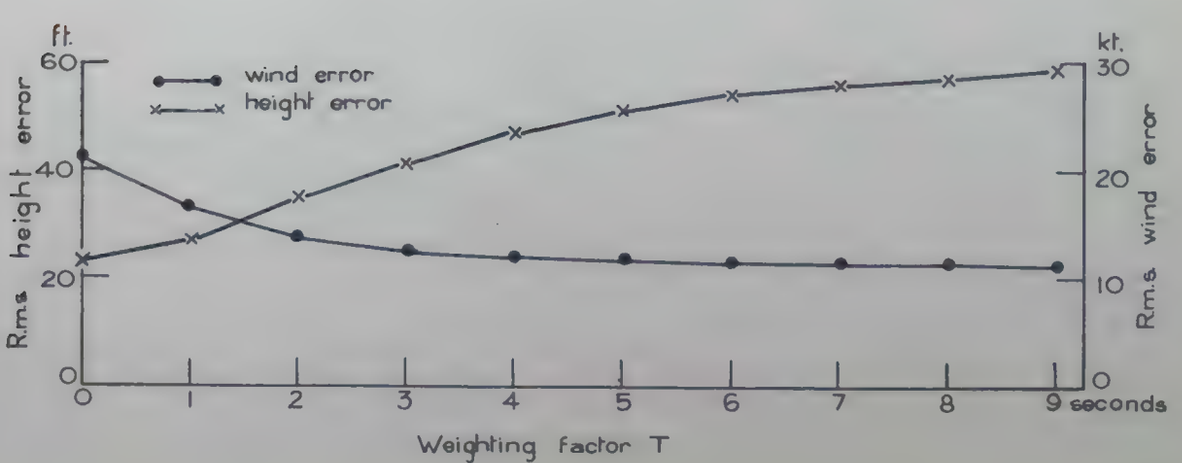


Fig. 6

COMPUTATION No. 3

Errors of representation at Liverpool

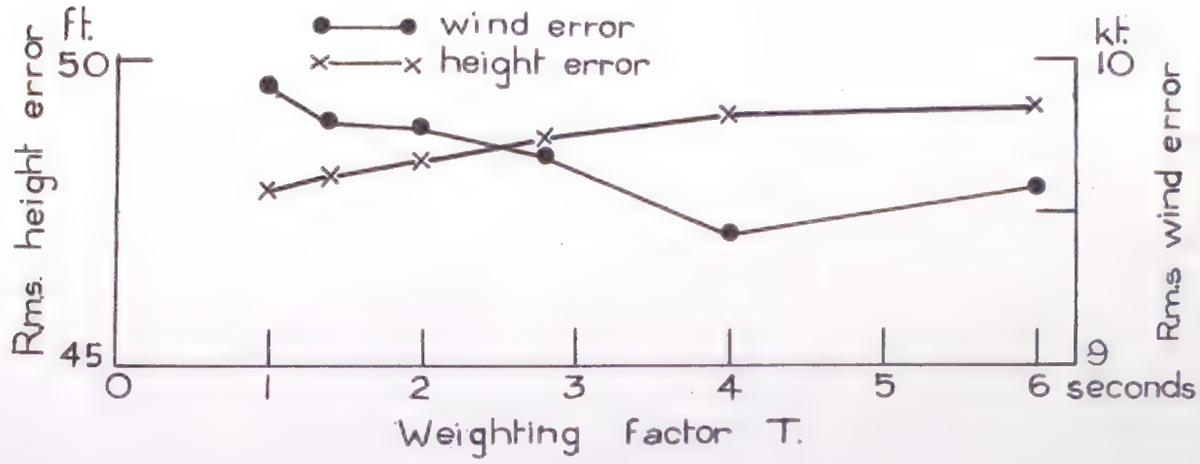


Fig. 7

COMPUTATION No. 3

Errors of representation at the six stations

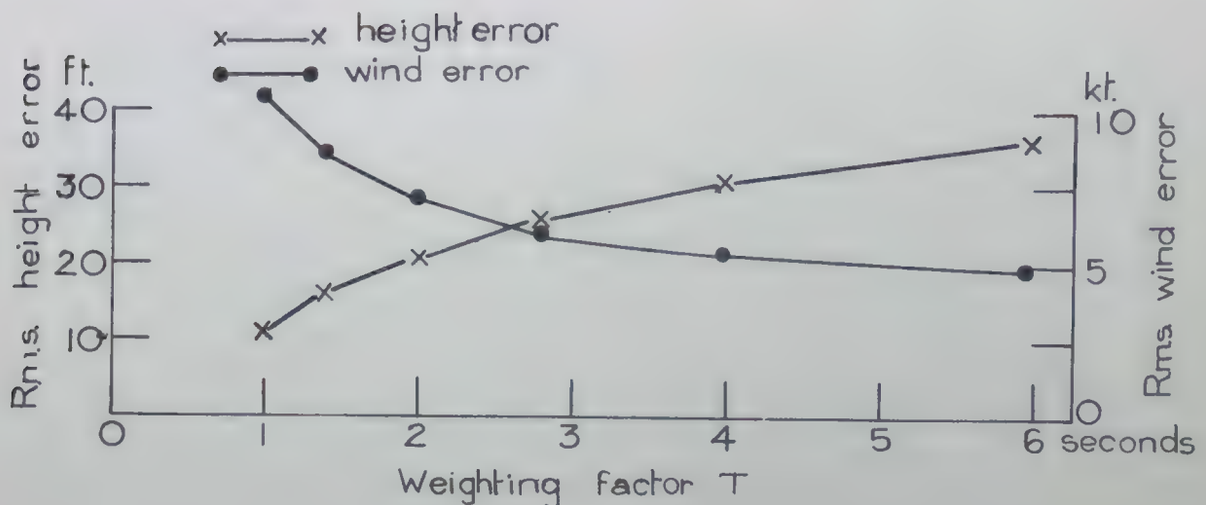


Fig. 8

COMPUTATION No. 5

Weighting factors

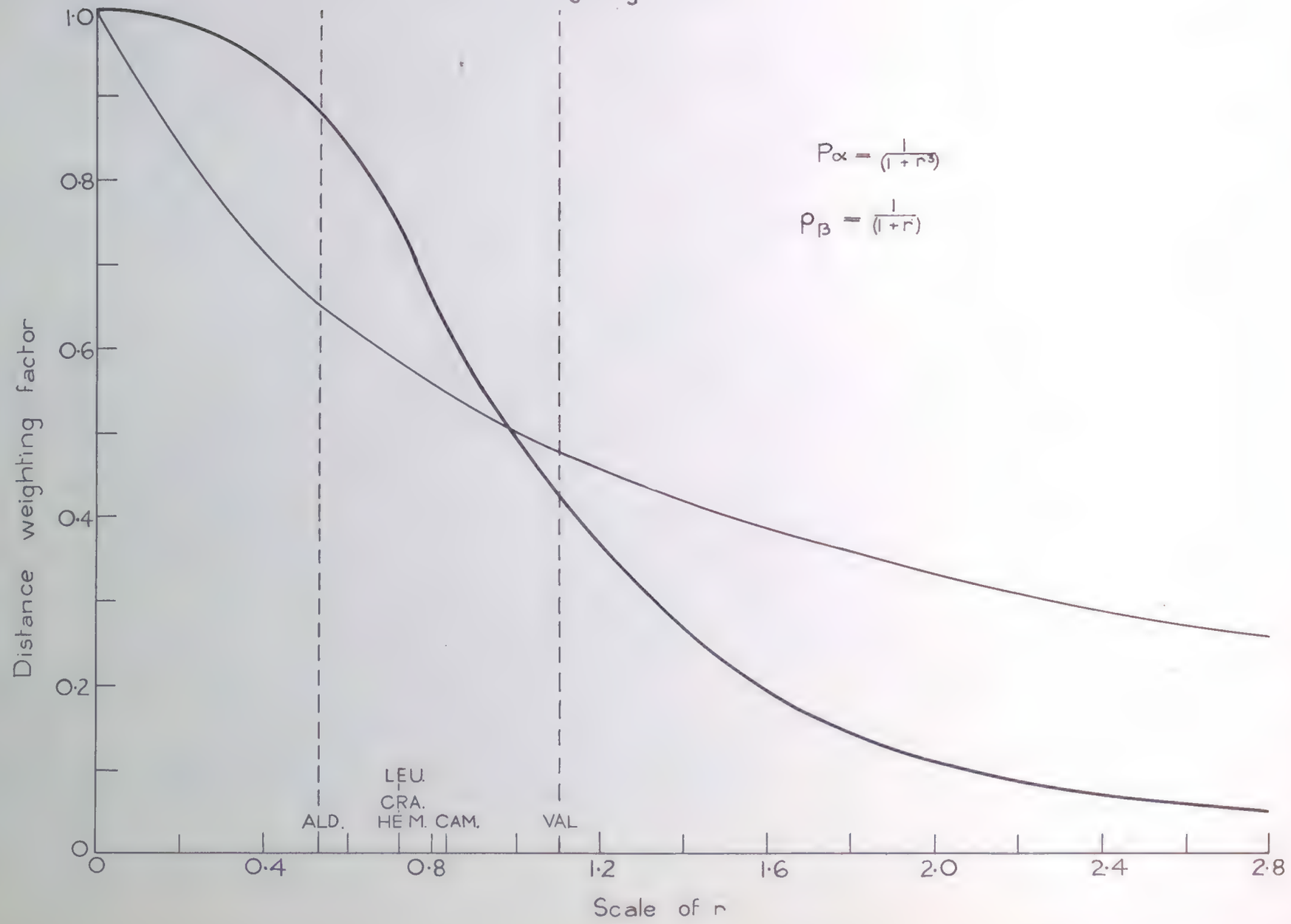
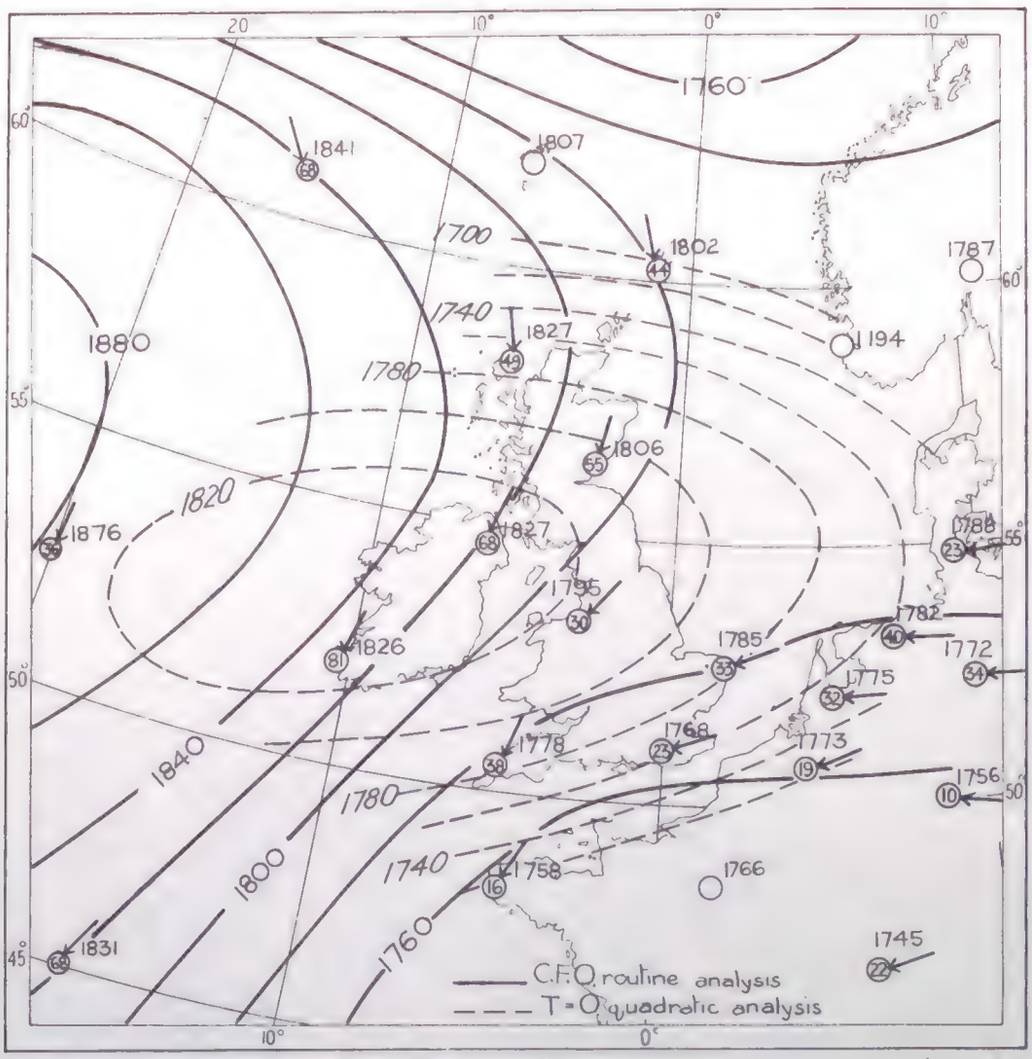


Fig. 9(a)

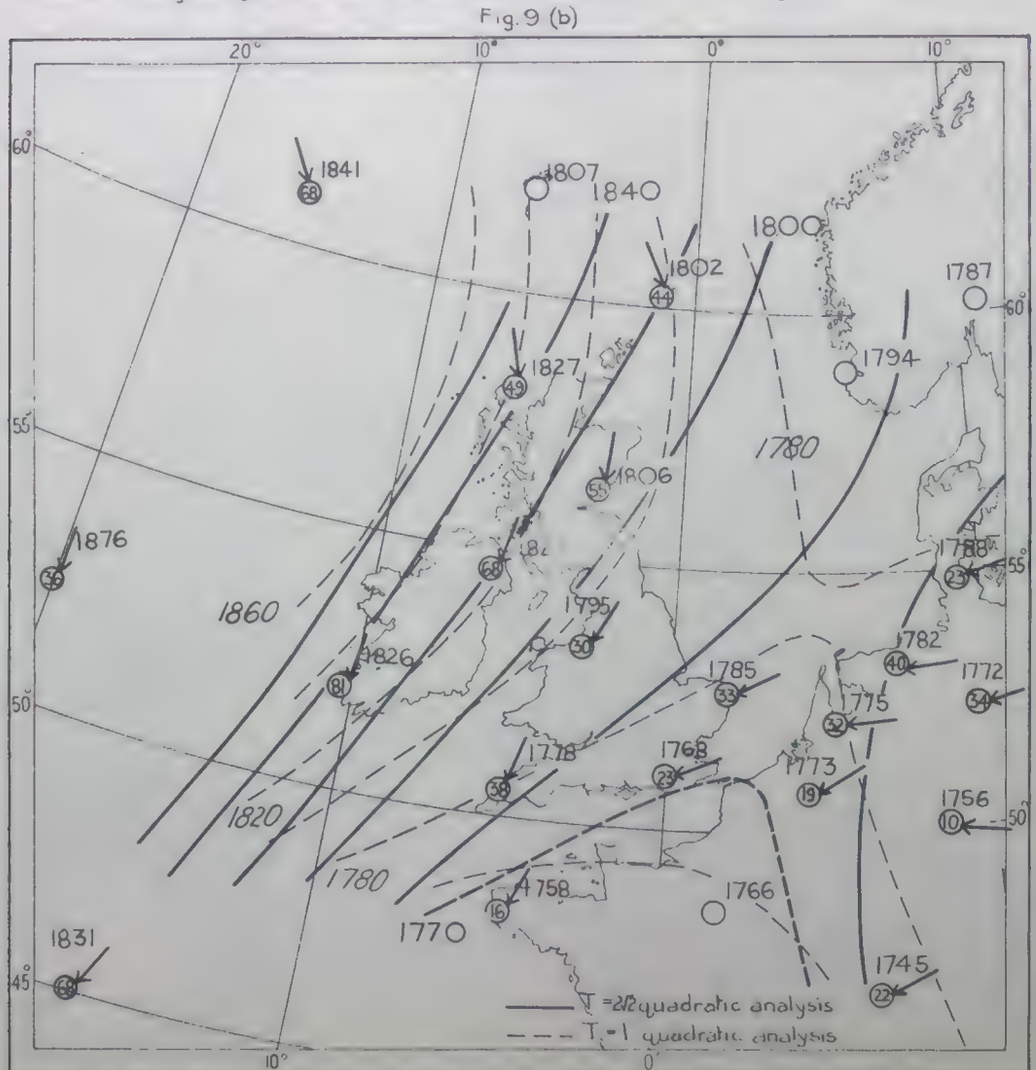
CONTOUR ANALYSIS — 500mb

1500 G.M.T. February 2nd 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 9 (b)



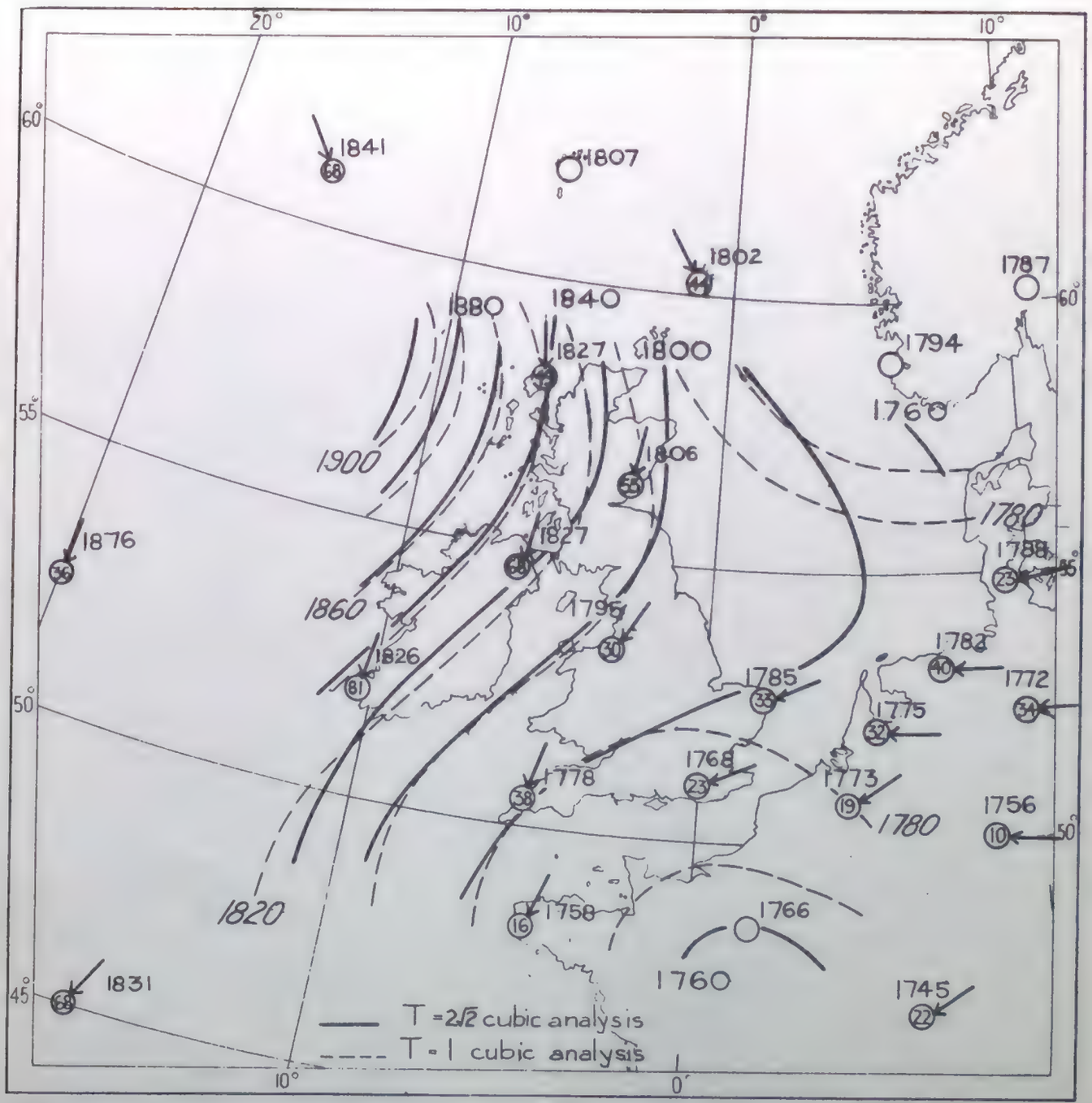
Observed heights given in tens of feet. Observed wind speed given in knots.

M.R.P. 965

Fig. 9 (c)

CONTOUR ANALYSIS — 500 mb.

1500 G.M.T. February 2nd. 1954

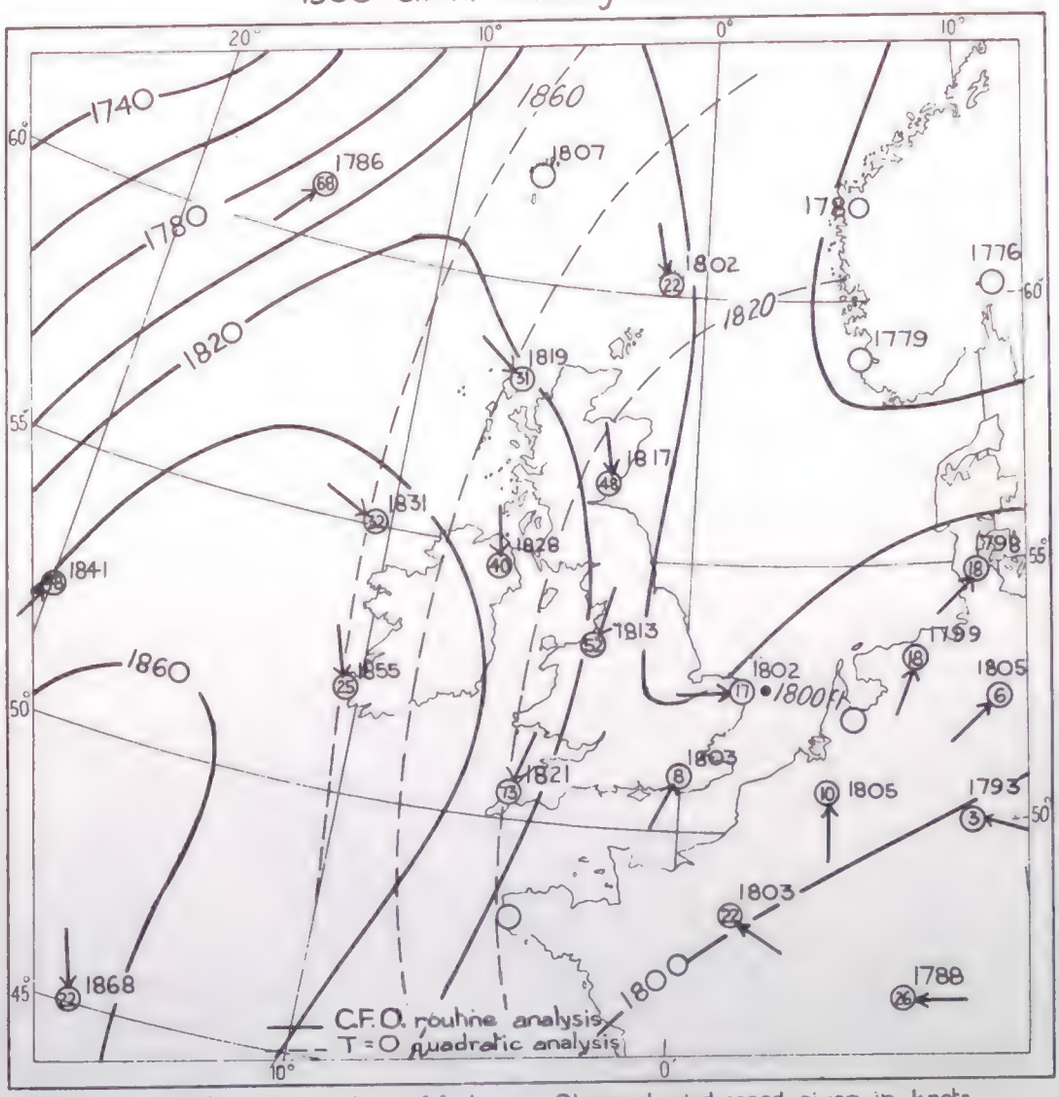


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 10 (a)

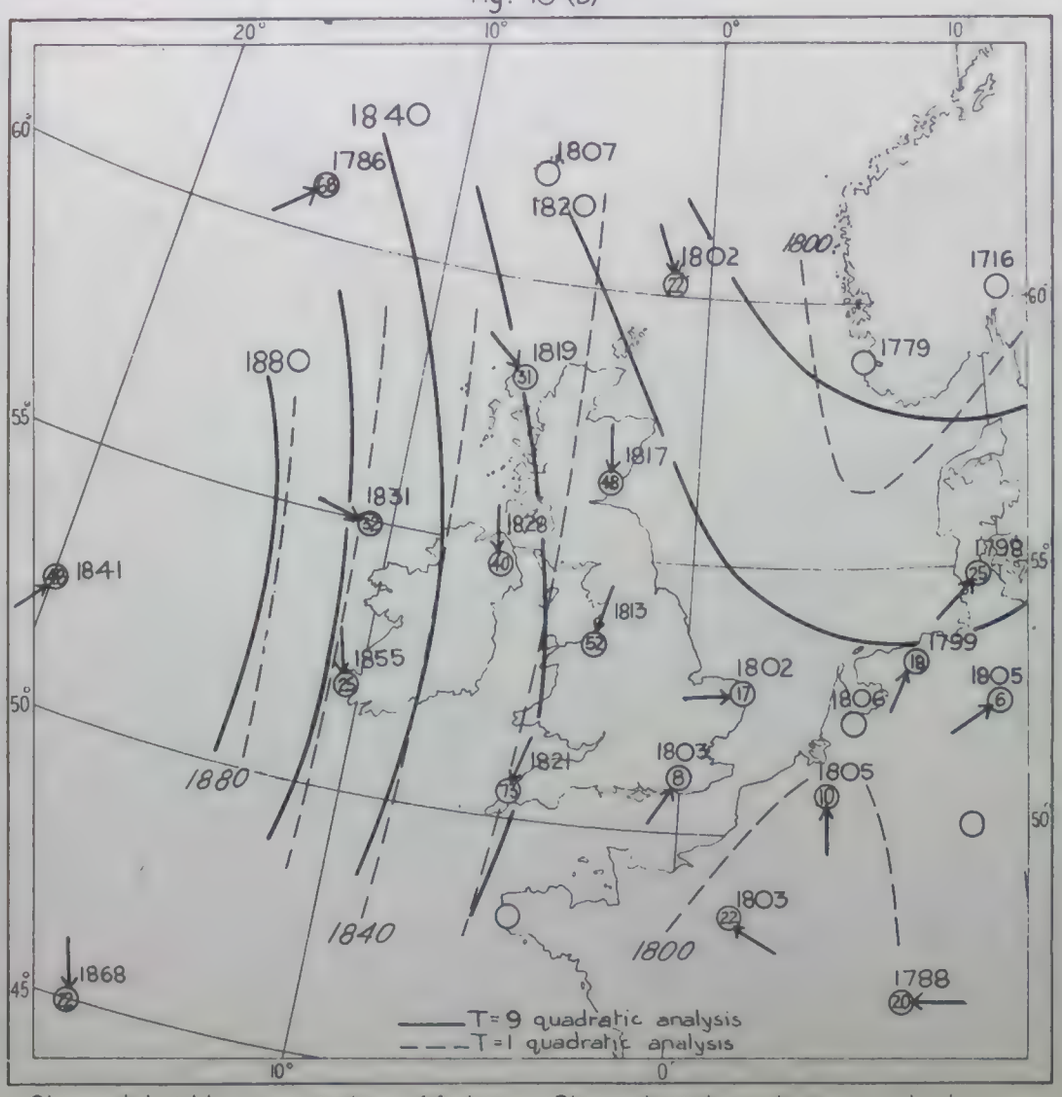
CONTOUR ANALYSIS – 500mb.

1500 G.M.T. February 15th 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 10 (b)

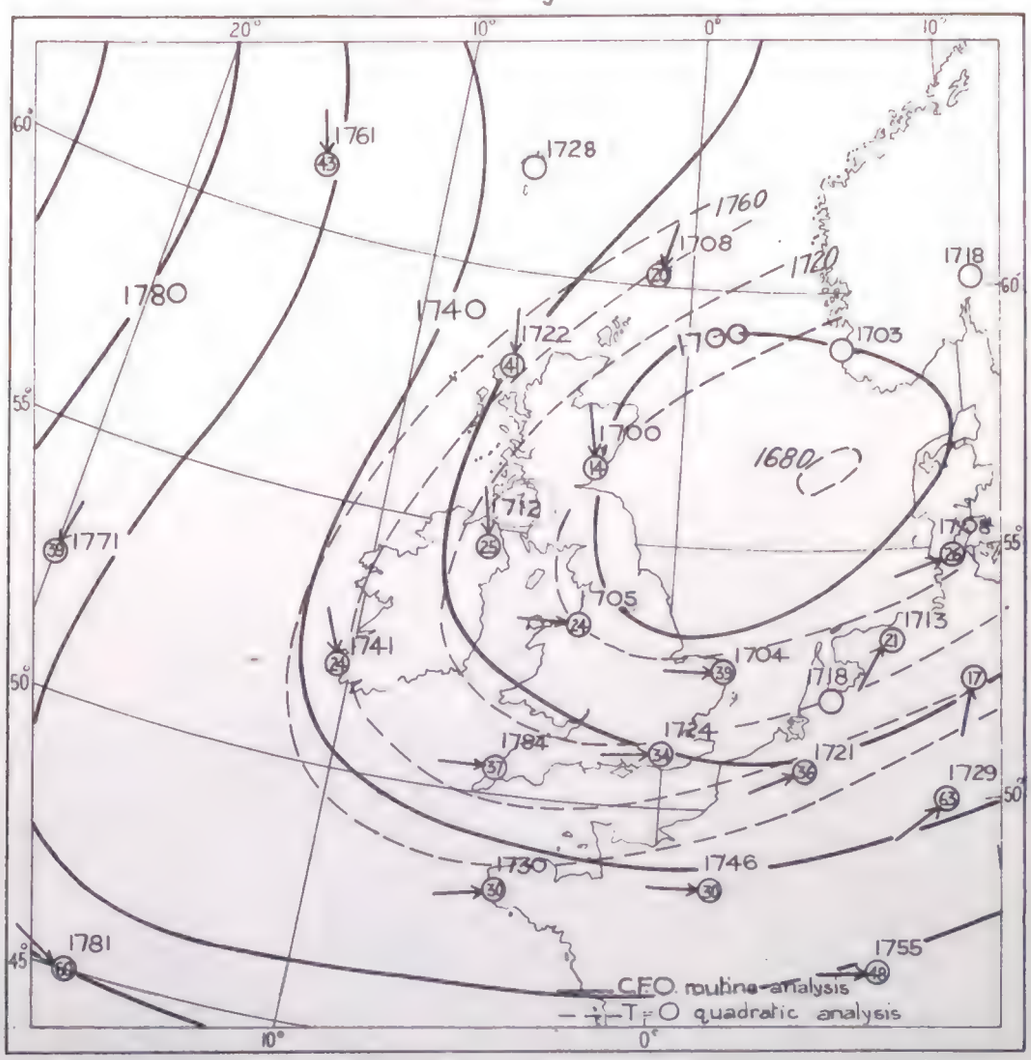


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 11 (a)

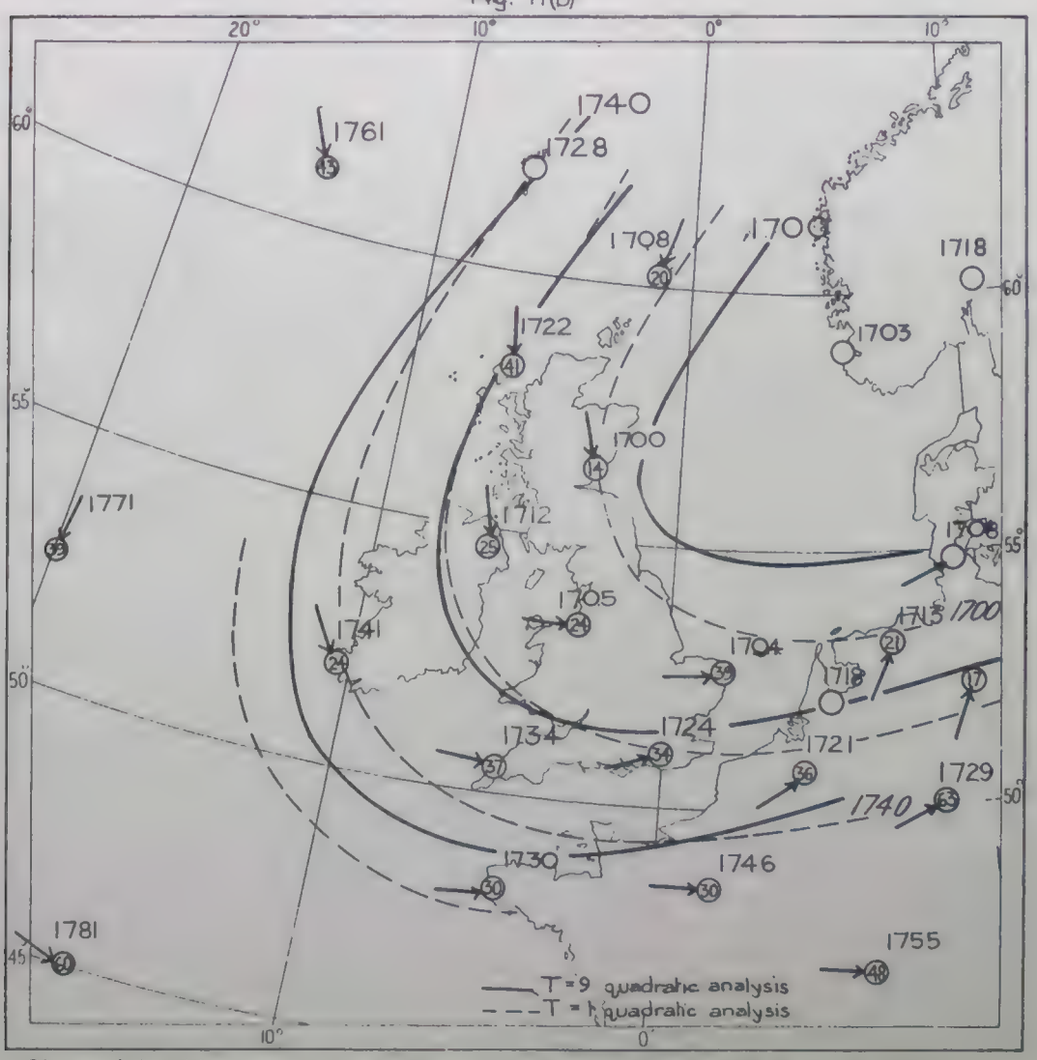
CONTOUR ANALYSIS – 500mb.

1500mb. February 27th 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 11(b)

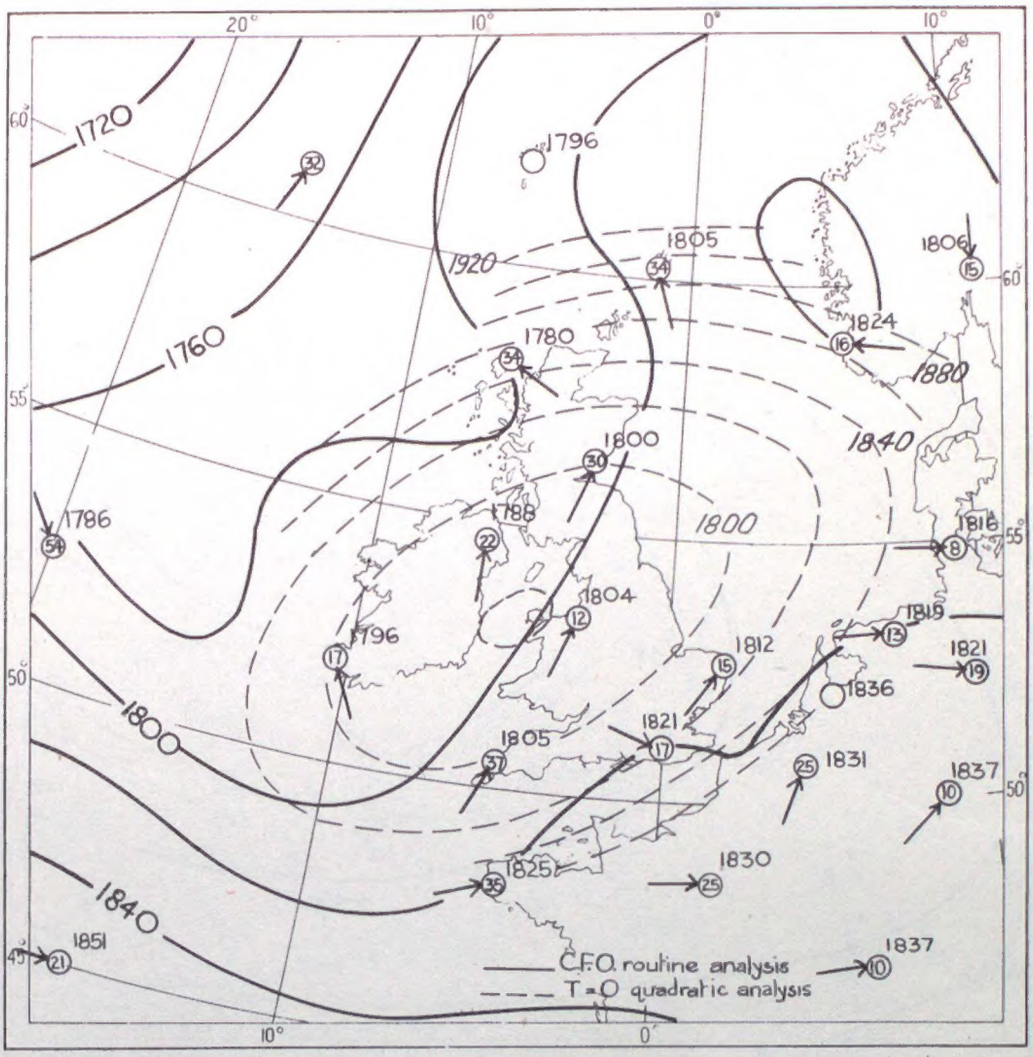


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 12(a)

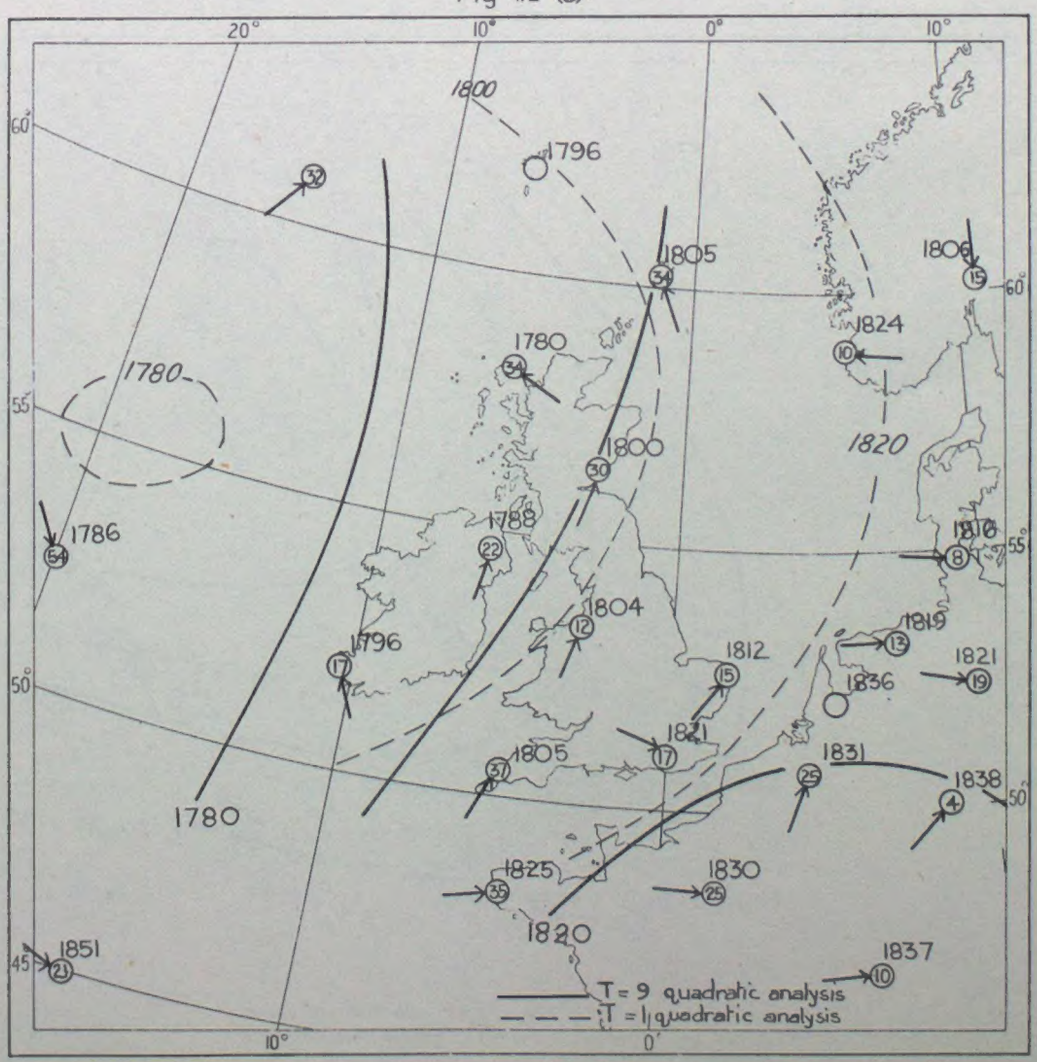
CONTOUR ANALYSIS—500mb.

1500 G.M.T. March 19th 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig .12 (b)

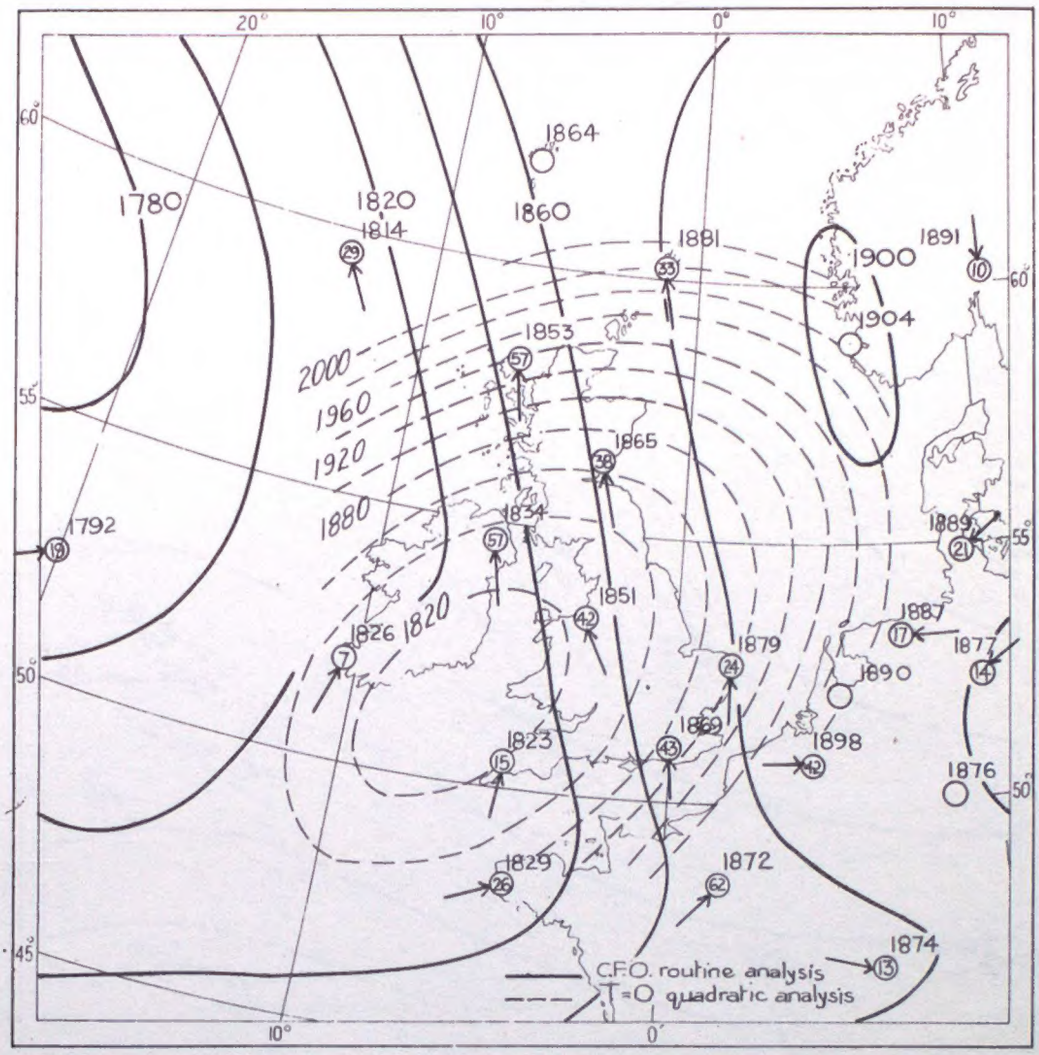


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 13(a)

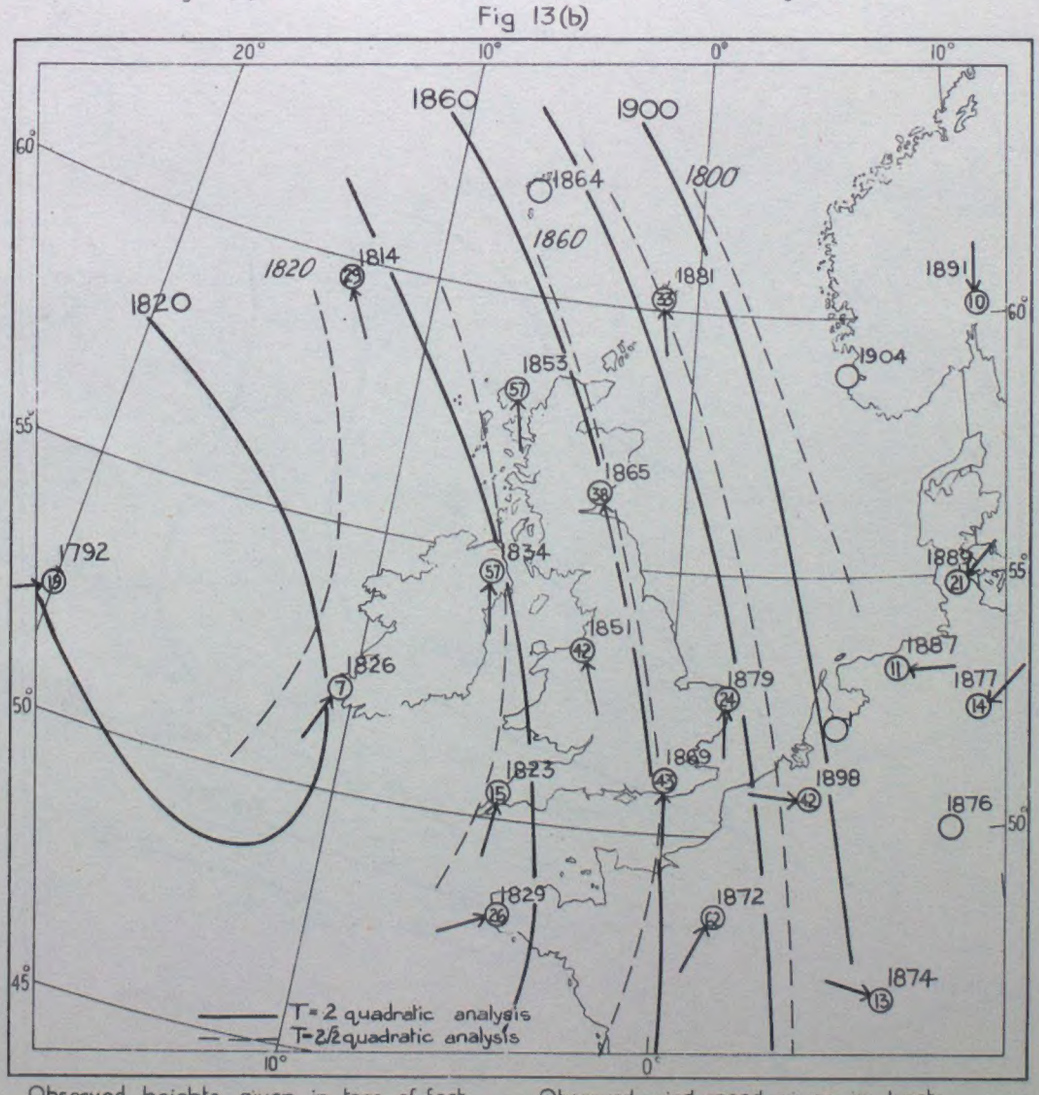
CONTOUR ANALYSIS — 500 mb

1500 G.M.T. May 28th. 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig 13(b)

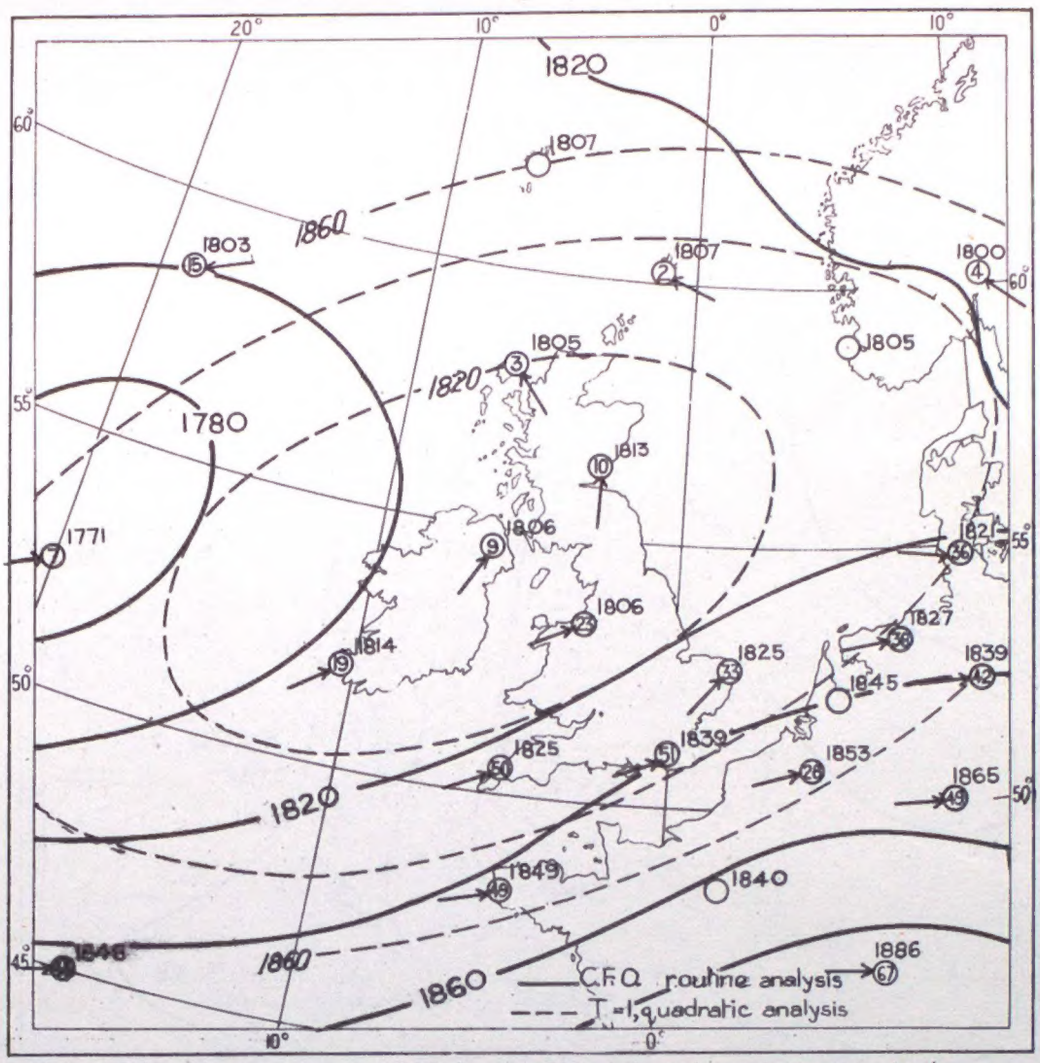


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 14-(a)

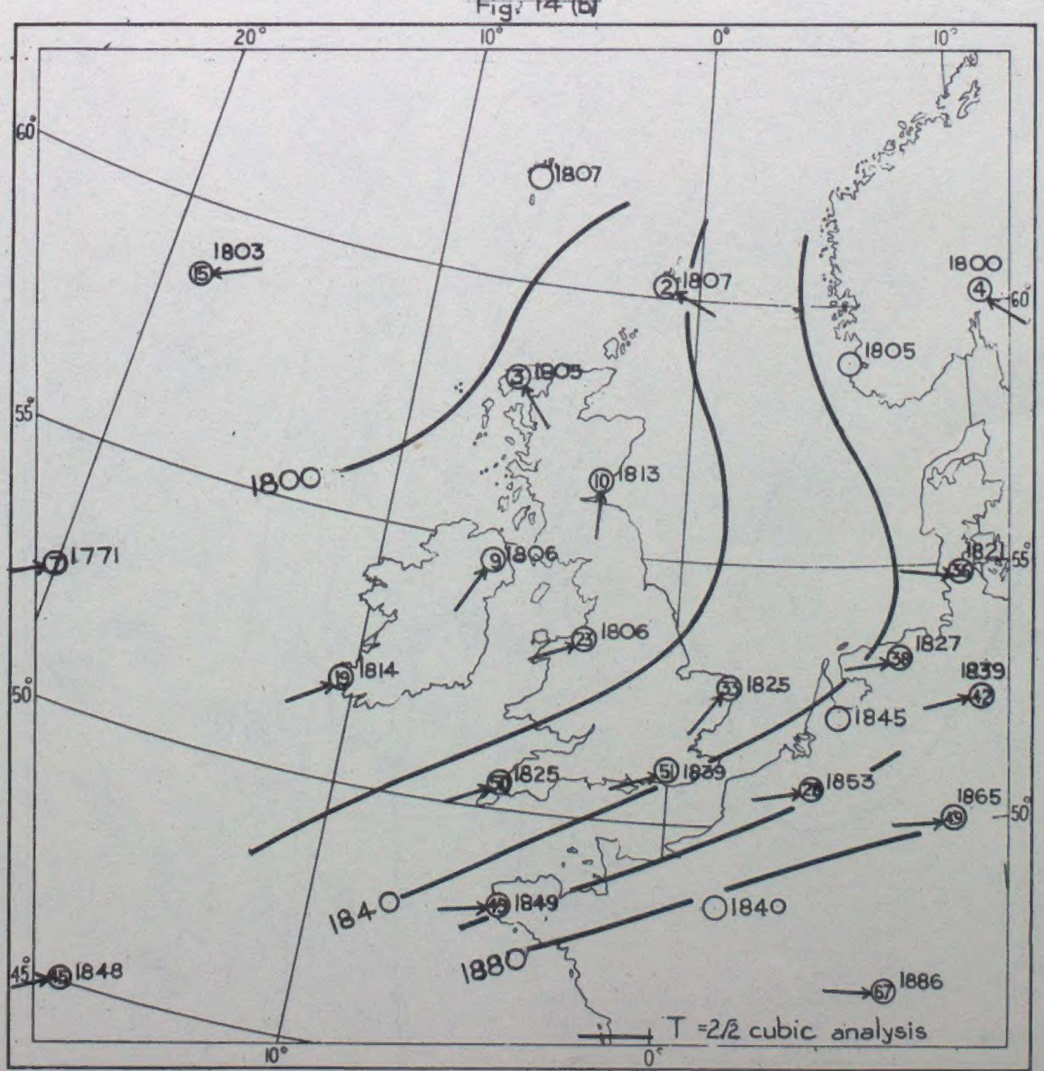
CONTOUR ANALYSIS — 500 mb.

1500 G.M.T. August 12 th. 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 14 (b)

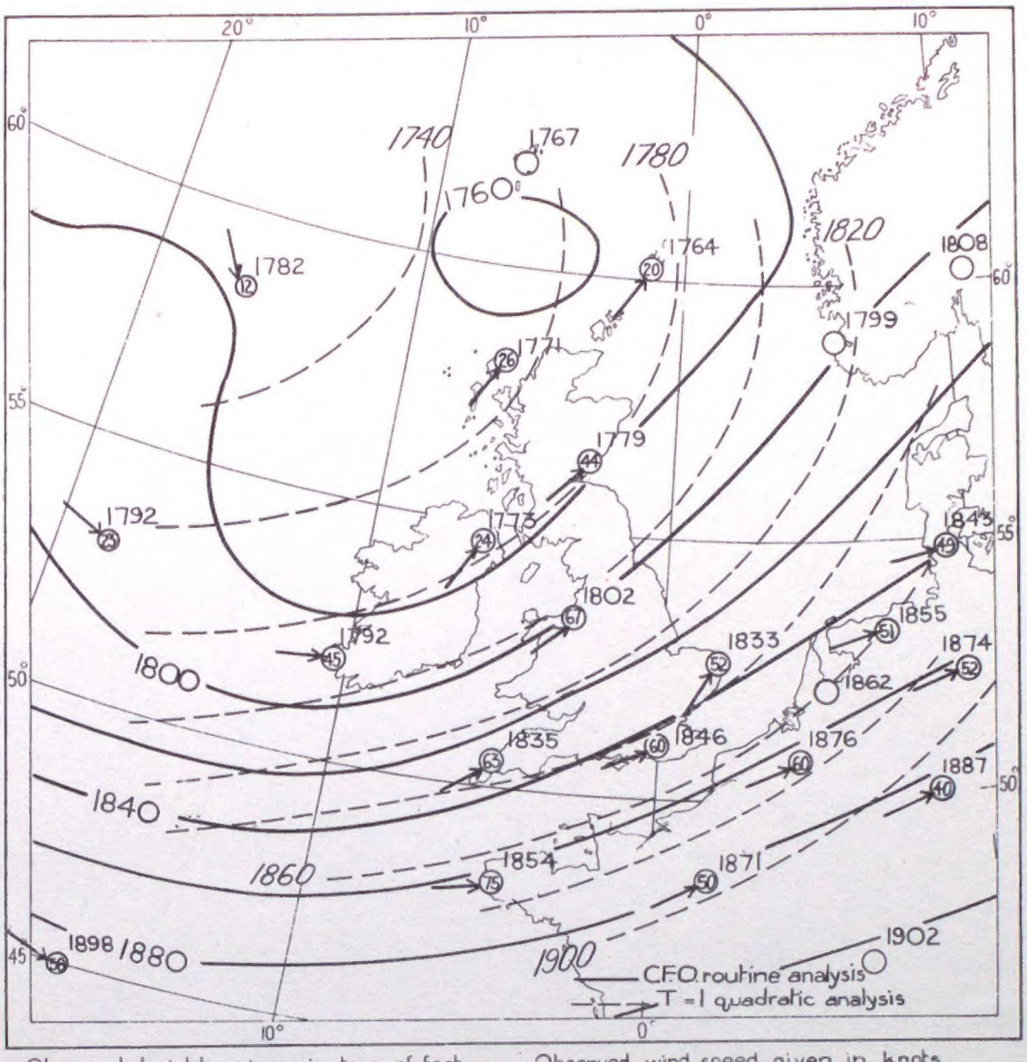


Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 15(a) M.R.P. 965

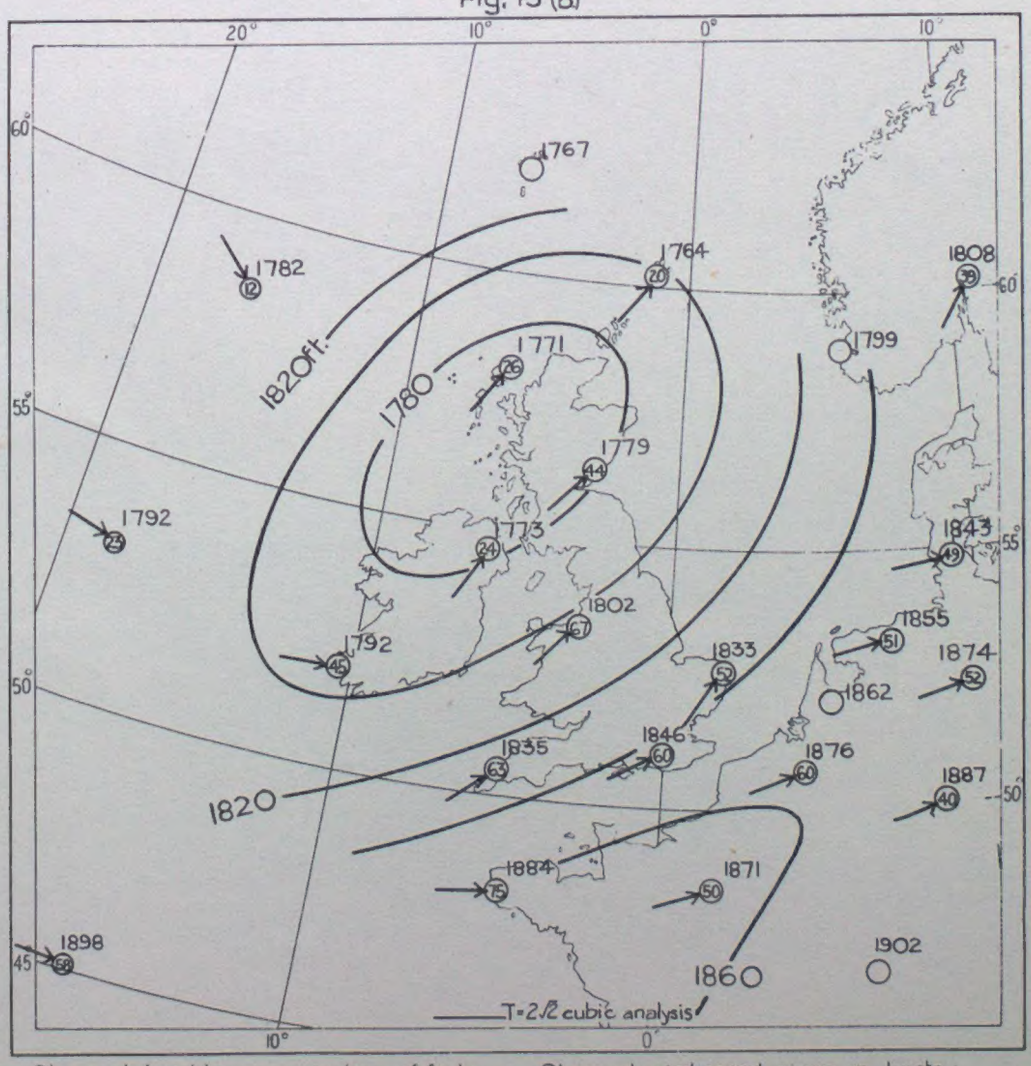
CONTOUR ANALYSIS — 500 mb.

1500 G.M.T. September 10th 1954



Observed heights given in tens of feet. Observed wind speed given in knots.

Fig. 15 (b)



Observed heights given in tens of feet. Observed wind speed given in knots.